

A Possible Explanation for the Radio Afterglow of GRB980519: The Dense Medium Effect

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ABSTRACT

GRB980519 is characterized by its rapidly declining optical and X-ray afterglows. Explanations of this behavior include models invoking a dense medium environment which makes the shock wave evolve quickly into the sub-relativistic phase, a jet-like outflow, and a wind-shaped circumburst medium environment. Recently, Frail *et al.* (1999a) found that the latter two cases are consistent with the radio afterglow of this burst. Here, by considering the trans-relativistic shock hydrodynamics, we show that the dense medium model can also account for the radio light curve quite well. The potential virtue of the dense medium model for GRB980519 is that it implies a smaller angular size of the afterglow, which is essential for interpreting the strong modulation of the radio light curve. Optical extinction due to the dense medium is not important if the prompt optical-UV flash accompanying the γ -ray emission can destroy dust by sublimation out to an appreciable distance. Comparisons with some other radio afterglows are also discussed.

Key words: gamma-rays: bursts – hydrodynamics: shock waves

1 INTRODUCTION

In the standard model of gamma-ray bursts (GRBs) (see Piran 1999 for a review), an afterglow is generally believed to be produced by the synchrotron radiation or inverse Compton scattering of the shock-accelerated electrons in an ultra-relativistic shock wave expanding in a homogeneous medium. As more and more ambient matter is swept up, the shock gradually decelerates while the emission from such a shock fades down, dominating at the beginning in X-rays and progressively at the optical to radio energy bands (Mészáros & Rees 1997; Waxman 1997a; Wijers & Galama 1999). In general, the light curves of X-ray and optical afterglows are expected to exhibit power-law decays (i.e. $F_\nu \propto t^{-\alpha}$) with the temporal index α in the range 1.1 – 1.4, given the energy spectral index of electrons $p \sim 2 - 3$. The observations of the earliest afterglows are in good agreement with this simple model (e.g. Wijers, Rees & Mészáros 1997; Waxman 1997b). However, over the past year, we have come to recognize a class of GRBs whose afterglows showed light curve breaks (e.g. GRB 990123, GRB 990510; Kulkarni *et*

al. 1999a; Harrison *et al.* 1999) or steeper temporal decays (i.e. $F_\nu \propto t^{-2}$; e.g. GRB 980519, GRB 980326; Bloom *et al.* 1999). Explanations for these behaviors include three scenarios: 1) a jet-like relativistic shock has undergone the transition from the spherical-like phase to a sideways-expansion phase (Rhoads 1999), as suggested by some authors (e.g. Sari, Piran & Halpern 1999; Kulkarni *et al.* 1999a; Harrison *et al.* 1999). 2) the shock wave propagates in a wind-shaped circumburst environment with the number density $n \propto r^{-2}$ (Dai & Lu 1998; Mészáros, Rees & Wijers 1998; Chevalier & Li 1999; Chevalier & Li 2000; Li & Chevalier 1999); 3) a dense medium environment ($n \sim 10^5 - 10^6 \text{ cm}^{-3}$) makes the shock wave evolve into the sub-relativistic phase after a short relativistic one (Dai & Lu 1999a,b). In the last model, since an afterglow from the shock at the sub-relativistic stage decays more rapidly than at the relativistic one, we will expect a light curve break or a long-term steeper decay, depending on the time when it begins to enter into the sub-relativistic stage. This scenario has reasonably interpreted the break in the R-band afterglow of GRB 990123 (Dai & Lu 1999a) and the steep decays of the X-ray and optical afterglows of GRB 980519 (Dai & Lu 1999b).

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Recently, Frail *et al.* (1999a) tried to test the first two

models (the jet and wind cases) by means of the radio afterglow behavior of GRB980519 and found that the wind model described it rather well. Due to the strong modulation of the light curve, however, they could not draw a decisive conclusion for the jet case. In this paper, we will examine the possibility of describing the evolution of the radio afterglow in terms of the dense medium model. Since this scenario involves the transition phase of the shock wave from the relativistic stage to the sub-relativistic, we have considered the trans-relativistic shock hydrodynamics in the numerical study. We first present the asymptotic result of the fitting of the radio data in section 2, and then the numerical result in section 3. In section 4, we show that the optical extinction due to the dense circumburst medium is not important, since the prompt optical-UV radiation, caused by the reverse shock, can destroy the dust by sublimation out to a substantial distance, as proposed by Waxman & Draine (1999). Finally, we give our discussions and conclusions.

2 ASYMPTOTIC BEHAVIOR OF THE RADIO AFTERGLOW IN THE SUB-RELATIVISTIC STAGE

GRB980519 is the second brightest GRB in the BeppoSAX sample. Its optical afterglow measured since ~ 8.5 hours after the burst exhibited rapid fading, consistent with $t^{-2.05 \pm 0.04}$ in *BVRI* (Halpern *et al.* 1999; Djorgovski *et al.* 1998), and the power-law decay slope of the X-ray afterglow, $\alpha_X = 2.07 \pm 0.11$ (Owens *et al.* 1998), was in agreement with the optical. The spectrum in optical band alone is well fitted by a power-law $\nu^{-1.20 \pm 0.25}$, while the optical to X-ray spectrum can also be fitted by a single power-law $\nu^{-1.05 \pm 0.10}$. The radio emission was observed with the Very Large Array (VLA) (Frail *et al.* 1999a) since about 7.2 hours after the burst and referred as VLA J232221.5+771543. The radio light curve shows a gradual rise to a plateau followed by a decline until below detectability after about 60 days. There are some large variations in these data, which is believed to be caused by interstellar scattering and scintillation (ISS; Frail *et al.* 1999a).

As discussed by Dai & Lu (1999b), the steep decays of the X-ray and optical afterglows of GRB980519 can be attributed to the shock evolution into the sub-relativistic phase 8 hours after the burst as the result of the dense circumburst medium. During such a sub-relativistic expansion phase, the hydrodynamics of the shocked shell is described by the self-similar Sedov-von Neumann-Taylor solution. The shell radius and its velocity scale with time as $r \propto t_{\oplus}^{2/5}$ and $\beta \propto t_{\oplus}^{-3/5}$, where t_{\oplus} denotes the time measured in the observer frame. Then, we obtain the synchrotron peak frequency $\nu_m \propto t_{\oplus}^{-3}$, the cooling frequency $\nu_c \propto t_{\oplus}^{-1/5}$, the peak flux $F_{\nu_m} \propto t_{\oplus}^{3/5}$ and the self-absorption frequency $\nu_a \propto t_{\oplus}^{-\frac{3p-2}{p+4}}$ for the case of $\nu_a > \nu_m$ (Dai & Lu 1999b). Now, the derived spectra and light curves are

$$F_{\nu} = \begin{cases} (\nu_a/\nu_m)^{-(p-1)/2} (\nu/\nu_a)^{5/2} F_{\nu_m} \propto \nu^{5/2} t_{\oplus}^{11/10}, & \text{if } \nu_m < \nu < \nu_a; \\ (\nu/\nu_m)^{-(p-1)/2} F_{\nu_m} \propto \nu^{-(p-1)/2} t_{\oplus}^{(21-15p)/10}, & \text{if } \nu_a < \nu < \nu_c; \\ (\nu_c/\nu_m)^{-(p-1)/2} (\nu/\nu_c)^{-p/2} F_{\nu_m} \propto \nu^{-p/2} t_{\oplus}^{(4-3p)/2}, & \text{if } \nu > \nu_c. \end{cases} \quad (1)$$

If the observed optical afterglow was emitted by slow-cooling electrons while the X-ray afterglow from fast-cooling electrons and if $p \approx 2.8$, then according to Eq.(1), the decay index $\alpha_R = (21 - 15p)/10 \approx -2.1$ and $\alpha_X = (4 - 3p)/2 \approx -2.2$, in excellent agreement with observations. Also, the model spectral index at the optical to X-ray band, $\beta = -(p-1)/2 \approx -0.9$ is quite consistent with the observed one -1.05 ± 0.10 . Furthermore, from the information of X-ray and optical afterglows, Dai & Lu (1999b) have inferred the physical parameters of this burst as follows:

$$\begin{aligned} E &\sim 0.3 \times 10^{52} \text{ erg}, & \epsilon_e &\sim 0.16, & \epsilon_B &\sim 2.8 \times 10^{-4}, \\ n &\sim 3 \times 10^5 \text{ cm}^{-3}, & z &\sim 0.55, \end{aligned} \quad (2)$$

where E is shock energy, z is the redshift of the burst and ϵ_e and ϵ_B are the electron and magnetic energy fractions of the shocked medium, respectively.

After the 60-day radio observational data being published, we promptly checked the dense medium model, and found that the asymptotic analysis can approximately describe the radio behavior. The analysis is as follows: adopting the inferred values of the physical parameters in Eq.(2), the detected frequency $\nu_* = 8.46 \text{ GHz}$ equals to ν_a at about day 12; thus, according to Eq.(1), we expect that before this time the radio emissions rise as $t_{\oplus}^{1.1}$ and then decay as $t_{\oplus}^{-2.1}$ after the self-absorption frequency ν_a falls below ν_* . This simple asymptotic solution agrees qualitatively with observations, as showed in Fig. 1 by the dotted line. This preliminary analysis stimulated us to fit the radio data with a more detailed model by taking into account the trans-relativistic shock hydrodynamics and the strict self-absorption effects of the synchrotron radiation.

3 TRANS-RELATIVISTIC SHOCK HYDRODYNAMICS, SELF-ABSORPTION EFFECT AND THE FITTING OF THE RADIO DATA

We consider an instantaneous release of a large amount of energy E in a constant density external medium. The energy released drives in the medium a shock wave, whose dynamic evolution from the relativistic to sub-relativistic phase can be described approximately in the following way.

Let r be the shock radius, γ and Γ be, respectively, the Lorentz factors of the shell and the shock front, β be the velocity of the shock front. As usual, the shock expansion is assumed to be adiabatic, during which the energy is conserved, and we have (Blandford & McKee 1976)

$$\frac{4}{3} \pi \sigma \beta^2 \Gamma^2 r^3 n m_p c^2 = E, \quad (3)$$

where σ is a coefficient: $\sigma \rightarrow 0.35$ when $\beta \rightarrow 1$ and $\sigma \rightarrow 0.73$ when $\beta \rightarrow 0$. As Huang, Dai & Lu (1998), we use an approximate expression for σ : $\sigma = 0.73 - 0.38\beta$.

The radius of the shock wave evolves as (Huang, Dai & Lu 1998)

$$\frac{dr}{dt_{\oplus}} = \beta c \gamma (\gamma + \sqrt{\gamma^2 - 1}) / (1 + z). \quad (4)$$

and the jump conditions of the shock are given by (Blandford & McKee 1976)

$$n' = \frac{\hat{\gamma}\gamma + 1}{\gamma - 1}n, \quad e' = \frac{\hat{\gamma}\gamma + 1}{\hat{\gamma} - 1}(\gamma - 1)nm_p c^2, \quad (5)$$

$$\Gamma^2 = \frac{(\gamma + 1)[\hat{\gamma}(\gamma - 1) + 1]^2}{\hat{\gamma}(2 - \hat{\gamma})(\gamma - 1) + 2}, \quad (6)$$

where e' and n' are the energy and the number densities of the shell in its comoving frame and $\hat{\gamma}$ is the adiabatic index, which equals 4/3 for ultra-relativistic shocks and 5/3 for sub-relativistic shocks. A simple interpolation between these two limits $\hat{\gamma} = \frac{4\gamma+1}{3\gamma}$ gives a valid approximation for trans-relativistic shocks (Dai, Huang & Lu 1999). Using the above equations, we can now numerically obtain the evolution of $r(t_\oplus)$ and $\gamma(t_\oplus)$ in the trans-relativistic stage, given proper initial conditions.

As usual, we assume that the distribution of relativistic electrons with the Lorentz factor γ_e takes a power-law form with the number density given by $n(\gamma_e)d\gamma_e = C\gamma_e^{-p}d\gamma_e$ above a low limit γ_{min} , which is determined by the shock velocity: $\gamma_{min} = \epsilon_e \frac{(p-2)}{(p-1)} \frac{m_p}{m_e} (\gamma - 1)$. Also, the energy densities of electrons and magnetic fields are assumed to be proportional to the total energy density e' in the comoving frame as $U'_e = \epsilon_e e'$ and $B'_\perp = (8\pi\epsilon_B e')^{1/2}$. Thus, from the standard theory of the synchrotron radiation (Rybicki & Lightman 1979; Li & Chevalier 1999), we have the expressions of the effective optical depth and the self-absorbed flux

$$\tau_{\nu'} = \frac{p+2}{8\pi m\nu'^2} \frac{\sqrt{3}q^3}{2mc^2} \left(\frac{4\pi m c \nu'}{3q}\right)^{-p/2} F_2\left(\frac{\nu'}{\nu'_m}\right) C B'_\perp^{(p+2)/2} \Delta r', \quad (7)$$

$$\nu'_m = \frac{3\gamma_{min}^2 q B'_\perp}{4\pi m c}, \quad C = (p-1)n'\gamma_{min}^{p-1}, \quad \Delta r' = r/\eta, \quad (8)$$

$$F_\nu = (1+z)D^3\pi\left(\frac{r}{d_L}\right)^2 \frac{2m\nu'^2}{p+2} \left(\frac{4\pi m c \nu'}{3q B'_\perp}\right)^{1/2} \frac{F_1(\nu'/\nu'_m)}{F_2(\nu'/\nu'_m)} (1-e^{-\tau}), \quad (9)$$

where $F_1(x)$, $F_2(x)$ are defined by Eq.(5) in Li & Chevalier (1999), m and q denote the mass and charge of the electron, d_L is the luminosity distance of the burst, assuming a flat Friedman universe with $H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\eta \sim 10$, characterizing the width of the shock shell. Here $D \equiv 1/[\gamma(1-\beta)]$ describes the relativistic effect, and ν relates to the corresponding frequency ν' in the comoving frame by $\nu = D\nu'/(1+z)$.

Using the above full set of equations, we computed the radio flux at the frequency $\nu_* = 8.46 \text{ GHz}$ and plotted the model fit in Fig. 1 as the solid line. We find that the following combination of the parameters fits almost all valid data rather well: $E \sim 0.8 \times 10^{52} \text{ erg}$, $\epsilon_e \sim 0.2$, $\epsilon_B \sim 1 \times 10^{-4}$, $n \sim 1 \times 10^5 \text{ cm}^{-3}$, $z \sim 0.55$ and the Lorentz factor at the initial time $t_\oplus = 1/3$ days, $\gamma = 1.2$ ($\beta \sim 0.55$). We stress that these parameters are in excellent agreement with those inferred independently from the X-ray and optical afterglows (Dai & Lu 1999b), as listed in Eq.(2). Clearly, there are some large amplitude variations in the observed light curve (e.g. about 18 days after the burst), which is believed to be caused by diffractive scintillation. Also plotted in Fig. 1 is the fit (dashed line) computed with the sub-relativistic model as presented in the appendix of Frail, Waxman & Kulkarni (1999). The fit with this model was obtained by adopting the initial conditions as: $t_0 = 1/3$ days and $r_0 = 1.6 \times 10^{16} \text{ cm}$ and the parameter values (ϵ_e , ϵ_B , E , z and n) the same as those in Dai & Lu (1999b). Comparing the trans-relativistic model with sub-relativistic one, we can easily see that the relativistic effect (as characterized by $D \equiv 1/\gamma(1-\beta)$ in Eq.(9)) flattens the rising phase at earlier time, making the trans-relativistic model agree better with the observations, while at the later time both the fitting curves trend towards the asymptotic solution (i.e. $F_\nu \propto t_\oplus^{-2.1}$).

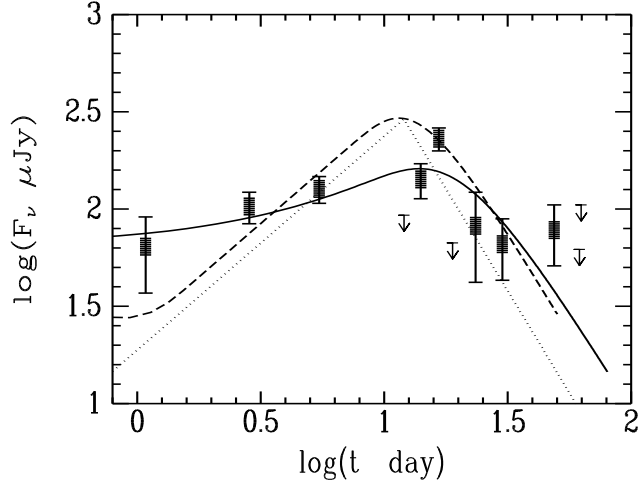


Figure 1. Model fits of the radio light curve at 8.46 GHz. Detections and upper limits for the non-detections, taken from Frail *et al.* (1999a), are indicated by the filled squares and arrows, respectively. The dotted line represents the expected asymptotic solution of the radio behavior according to the dense medium model in Dai & Lu (1999b). The solid and dashed lines represent the fits with the trans-relativistic and sub-relativistic models, respectively. See text for additional details.

4 DUST SUBLIMATION AND OPTICAL EXTINCTION BY DENSE MEDIUM

One may ask whether the dense circumburst medium may cause large extinction in the optical afterglow of GRB 980519. A crude estimate is as follows. At the time that the blast wave transits to the sub-relativistic stage ($t_\oplus \sim \frac{1}{3}$ days, $\beta \sim 0.55$, $\gamma \sim 1.2$), from Eq.(3) we derived its radius to be $r \sim 2.1 \times 10^{16} \text{ cm}$. Therefore the characteristic column density through the medium into which the blast wave is expanding is about $nr \sim 2.1 \times 10^{21} \text{ cm}^{-2}$ with the corresponding A_V of 1.3 magnitudes in the rest frame of the absorber. This column density is comparable to (but slightly larger than) the Galactic 21cm column density ($\sim 1.74 \times 10^{21} \text{ cm}^{-2}$, Halpern *et al.* 1999) in the direction of the GRB 980519. With A_λ scales linearly with $\frac{1}{\lambda}$, the absorption in our observed B band for this optical transient is about $1.3(1+z) \sim 2$ magnitudes, twice the value adopted by Halpern *et al.* (1999) for correction of the relative extinction.

In the above estimate, we have made a questionable assumption, i.e. the dense medium around the burst has a standard gas-to-dust ratio. However, this may be not realistic, considering that the dust around the burst can be destroyed due to sublimation out to an appreciable distance (\sim a few pc) by the prompt optical-UV flash (Waxman & Draine 1999; hereafter WG99), accompanying the prompt burst. Below we will give an estimation of the destruction radius for dense medium case, following WG99.

The prompt optical flash detected accompanying GRB 990123 (Akerlof *et al.* 1999) suggests that, at least for some GRBs, γ -ray emission is accompanied by prompt optical-UV radiation with luminosity in the 1-7.5eV range of the order

of $10^{49}(\frac{\Delta\Omega}{4\pi})\text{erg/s}$ for typical GRB parameters, where $\Delta\Omega$ is the solid angle into which γ -ray and optical-UV emission is beamed (WG99). The most natural explanation of this flash is emission from a reverse shock propagating into the fireball ejecta shortly after it interacts with the surrounding gas (Sari & Piran 1999; Mészáros & Rees 1999). As for GRB 980519 with the parameter values as $\epsilon_e \sim 0.2$, $\epsilon_B \sim 10^{-4}$, $n \sim 10^5\text{cm}^{-3}$, $E \sim 8 \times 10^{51}\text{erg}$ and the burst duration $\Delta t \sim 70\text{s}$, we derive the luminosity in the 1-7.5 eV range to be about $L_{1-7.5} \sim 5 \times 10^{48}\text{erg/s}$ (Here we have assumed that electron and magnetic field energy fractions in the reverse shock are similar to those in the forward shock; Wang, Dai & Lu 1999c). The condition for the grain to be completely sublimed during the prompt flash time is

$$T > T_c \simeq 2300\text{K}[1 + 0.033\ln(\frac{a-5}{\Delta t/10\text{s}})], \quad (10)$$

where T is the grain temperature, determined by Eq.(8) of WG99, and $a \equiv a_{-5} \times 10^{-5}\text{cm}$ is the radius of the dust grain. Then, according to Eq.(17) of WG99, the radius out to which the prompt flash can heat grains to the critical temperature T_c is

$$R_c \simeq 3.7 \times 10^{19} (\frac{Q_{UV} L_{49}(1 + 0.1a_{-5})}{a_{-5}})^{1/2} \text{cm} \simeq 2.7 \times 10^{19} \text{cm}, \quad (11)$$

where Q_{UV} is the absorption efficiency factor of the optical-UV flash and can be assumed to be near one for grain radii $a > 10^{-5}$ expected in dense medium. Thus, we can safely say that the extinction due to the circumburst dense medium is not important if the size of the dense medium is shorter than R_c and we think that this condition is reasonable for GRB 980519, and also for GRB 990123 (Dai & Lu 1999a).

5 DISCUSSIONS AND CONCLUSIONS

The detection of strong diffractive scintillation requires that the angular size of the source VLA J232221.5+771543 should be less than $1\mu\text{arcsec}$ even 15 days after the burst (Frail *et al.* 1999a), otherwise the fluctuations would be suppressed. This small inferred size is not consistent with the spherical, homogeneous model with a normal density ($n \sim 1\text{cm}^{-3}$), but can marginally be consistent with the jet model and the wind-shaped circumburst medium model. We note that the dense medium model may have a potential advantage for this requirement, simply because that the shock will be quickly decelerated to a sub-relativistic velocity and therefore have a shorter shock radius. This can be clearly seen from the following comparison: $\theta_{s,rel} \propto \int_0^{t_\oplus} 2\gamma^2 c dt (1/\gamma(t_\oplus))$ for the relativistic case while $\theta_{s,sub} \propto \int_0^{t_\oplus} \beta c dt (\beta < 1)$ for the sub-relativistic one. Here, for the afterglow of GRB 980519, we assume that before $t_\oplus \sim 1/3\text{days}$, the shock is adiabatic and relativistic; thus the shock radius is $r(t) \simeq (17Et_\oplus/4\pi m_p n c(1+z))^{1/4} \simeq 0.78 \times 10^{16} (E/3 \times 10^{51}\text{erg})^{1/4} (n/3 \times 10^5\text{cm}^{-3})^{-1/4} (t_\oplus/1/3\text{days})^{1/4} [(1+z)/1.55]^{-1/4} \text{cm}$ (Sari, Piran & Narayan 1998). Then, the shock radius should follow the Sedov-von Neumann-Taylor self-similar solution as $r(t_\oplus) \propto t_\oplus^{2/5}$. Thus, we obtained the angular size of the afterglow $\theta_s \simeq 0.8\mu\text{arcsec}(t_\oplus/15\text{days})^{2/5}$. The agreement would be even improved if we note that at the beginning of the assumed sub-relativistic stage, the radius should increase more slowly than the self-similar solution in the trans-relativistic regime.

The strong modulations caused by scintillation also make the estimate of the spectral slope in the radio band less accurate. The *averaging* value of the spectral slope from day 12 on is $\beta \simeq -0.45 \pm 0.6$ (where $F_\nu \propto \nu^\beta$), implying that the time averaged self-absorption frequency ν_a is between 1.43 GHz and

4.86 GHz (Frail *et al.* 1999b). In our model, the time ($t_\oplus \simeq 12\text{days}$) when the fitting curve begins to decline corresponds to $\nu_a = \nu_* = 8.46\text{GHz}$. Since the self-absorption frequency decays as $\nu_a \propto t_\oplus^{-(3p-2)/(p+4)} \propto t_\oplus^{-0.94}$ for $p = 2.8$, we expect that ν_a shifted quickly below 4.86 GHz at day 21, but was above 1.43 GHz over all the detecting time, which is in reasonable agreement with the observations.

The radio afterglow of GRB 970508, the longest light curve (450-day) obtained by far, exhibited different behavior from GRB 980519 (Frail *et al.* 1997; Waxman, Kulkarni & Frail 1998; Frail, Waxman & Kulkarni 1999). From the spectral and temporal radio behavior, Frail, Waxman & Kulkarni (1999) inferred that the fireball has undergone a transition to sub-relativistic expansion at $t \sim 100\text{days}$, consistent with the inferred low ambient density $n \sim 1\text{cm}^{-3}$ (but also see Chevalier & Li 1999a). On the other hand, some radio afterglows (e.g. GRB 990510, GRB 981226; Frail *et al.* 1999b) show similar behaviors to GRB 980519, that is, they exhibit a slow rise to the maximum for a relatively short time and then a fast decline until below detectability. It is likely that the shocks of these bursts entered into the sub-relativistic stage after a short relativistic one and our above model can also describe their radio afterglows. Harrison *et al.* (1999) had interpreted the broad-band lightcurve break in the afterglows of GRB 990510 as due to a jet-like outflow. We speculate that another possible explanation is that the shock had entered into the sub-relativistic stage after $\sim 1\text{days}$ as the result of the combination of the dense medium and jet effects (Wang *et al.* 1999b), the latter of which may be real in consideration of the large inferred isotropic energy. The radio afterglow of GRB 990123 is unique for its “flare” behavior (Kulkarni *et al.* 1999b), whose most natural explanation is that it arises from the reverse shock, as evidenced by the prompt optical flash (Sari & Piran 1999). Our preliminary computation (using the trans-relativistic model) shows that the radio emission from the forward shock in the dense medium model is significantly lower than that from the reverse shock and declines quickly after the peak time, if a jet-like outflow with an opening angle $\theta \sim 0.2$, as required by the “energy crisis” of this burst, is invoked (Wang *et al.* 1999b). Moreover, the fast decline of the radio emission from the forward shock, which is caused by the deceleration of the shock in the sub-relativistic stage, can be consistent with the non-detection even 3 days after the burst.

In summary, we argue that the dense medium model, which has interpreted the optical to X-ray afterglows of GRB 980519 quite well, can also account for the radio afterglow excellently. The circumburst environment can affect the evolution of GRBs afterglows significantly (Mészáros, Rees & Wijers 1998; Panaitescu, Meszaros & Rees 1998; Wang *et al.* 1999a). For a low ($n \sim 1\text{cm}^{-3}$), homogeneous density environment, the shock waves stay at the relativistic shock stage for quite a long time, while for the dense medium case, the shock wave quickly enters into the sub-relativistic stage. Recently, a generic dynamic model for the evolution of shocks from ultra-relativistic phase to the sub-relativistic one has been also developed by Huang *et al.* (1999a). The afterglows of the optically thin radiation (e.g. optical and X-rays) from the shock at the sub-relativistic stage decays more rapidly than at the relativistic one. As for the radio afterglow (usually $\nu_m < \nu_a$ at the sub-relativistic stage for this model), the dense medium model predicts a slow rise ($\nu_* < \nu_a$), followed by a round peak and a late steep decline ($\nu_* > \nu_a$), trending towards the behavior of the optical and X-ray afterglows. Clearly, this behavior is different from the jet model in the early epoch. But it is somewhat similar to the wind model, making it difficult to distinguish between them through the radio observations.

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